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Undergraduate teaching and learning of mathematics with open source textbooks: Uso de textos universitarios de matemáticas

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Resumen

In the Undergraduate Teaching and Learning in Mathematics with Open Software and Textbooks project we study the use of open source computational resources in the teaching and learning of mathematics at undergraduate level (Beezer et al., 2018). The project gathers (a) real-time, individualized viewing data from three dynamic undergraduate textbooks for calculus, linear algebra, and abstract algebra; (b) ongoing surveys of users' descriptions of the textbook use; (c) users' questionnaires (beliefs and attitudes towards mathematics, technology, teaching, and learning); and (d) student performance (tests of knowledge and grades). The textbooks have been enhanced with WeBWorK, Geogebra, and interactive Reading Questions for which instructors can see the responses in real time, and computational cells. In this article I highlight features of the textbooks and the theoretical and methodological approaches to answer two questions: How do students and instructors use textbooks? and How can we develop textbooks that will improve teaching and learning?

Palabras clave: open-source textbooks, textbook use, data analytics, calculus, linear algebra, abstract algebra

Within the array of resources for teaching and learning, the textbook continues to be the most prevalent one for instructors and students. Textbook formats have been changing from paper to digital, open source formats, including sophisticated tools such as computing cells, annotation tools, and powerful search engines, easing access at relatively low cost. Importantly, open source textbooks never expire or go out of print and can be distributed at no cost to students, making them practically fully accessible. The study we report here is part of a large U.S. funded project that seeks to describe how instructors and students use three open-source, technologically enhanced textbooks: Active Calculus (Boelkins, 2018), Linear Algebra (Beezer, 2017), and Abstract Algebra (Judson, 2017). These textbooks have been created in PreTexT, a markup language that allows for the textbooks to be viewed in any device and in any platform.

We use Rezat and Strässer's (2012) didactical tetrahedron to investigate how resources support instruction (Cohen, Raudenbush, & Ball, 2003, which is depicted in the base of the tetrahedron in Figure 1).

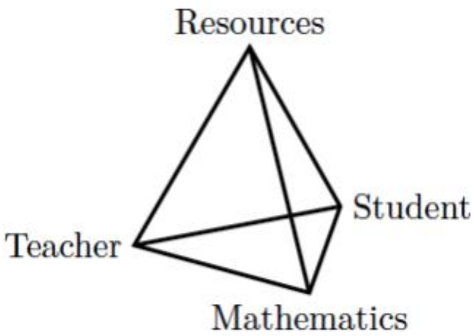

Figure 1. Resources can mediate instruction; the didactical tetrahedron (Rezat & Strässer, 2012, p. 241)

We follow Gueudet and Trouche (2009) in their definition of documents: the combination of a set of resources plus the schemes of utilization. Resources are defined as the collection instruments gathered for a particular purpose (e.g., textbook, past lecture notes, syllabi). Schemes of utilization include the processes that users engage in as they use the resources. These schemes have three distinct components, a material component (how the physical textbook or software is manipulated), a mathematical component (e.g., how the mathematical definitions are changed from canonical definitions), and the didactical component (e.g., how specific features are used). We seek to describe two documentation processes, *instrumentation*, that considers the influences on the user of the set of available resources, and *instrumentalization*, how the users change the resources as they use them (see *Figure 2*).

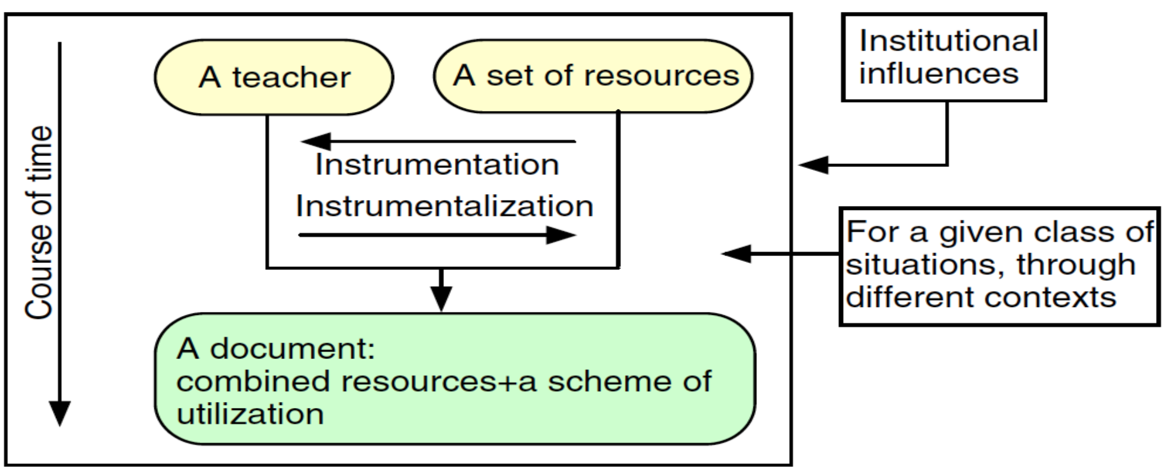

Figure 2: The documentational approach (Gueudet and Trouche 2009).

We do these by attending to two areas of instructors' work, lesson planning and its enactment seeking to identify operational invariants, instructors' beliefs that shape the design and use of resources (e.g., beliefs about ways with which students better understand definitions).

Methods

We use a mixed method design to gather use data from students and instructors as they engage with the textbooks (see *Figure 3*).

	Beginning of term	Week in the term						End of term
		2	4	6	8	10	12	14
Teacher surveys	X							
Teacher logs		X	X	X	X	X	X	
Course syllabi	X							
Computer-generated data of teacher and student textbook viewing								
Student logs		X	X	X	X	X	X	
Student survey					X			
Student tests	X							X

Figure 3. Data collection over a term.

Instructors and students fill out surveys at different points in time to describe their beliefs and attitudes towards mathematics and technology. We collect tests of students' knowledge at the beginning and at the end of the semester to gather information about their knowledge growth. In addition we collect student and instructor logs (online surveys with four to seven questions about the use of the textbook during the past two weeks). In addition we collect computer generated viewing data (see *Figure 4*) which can be navigated at the user level (see *Figure 5*), time spent and number of clicks done on each textbook section and element (see *Figure 6*).

To analyze the data we use ongoing natural language processing (Blei, Ng, & Jordan, 2003) to gather themes from all the student responses to logs. Instructors' responses are analyzed manually, to identify the schemes of use of their textbooks. We aggregate across semesters to identify recurring themes and triangulate the log and viewing data with the time data to corroborate themes and patterns of viewing.

Results

I report briefly on findings from (1) the analysis of bi-weekly log data from 102 students from four instructors in four different states who were using a dynamic linear algebra textbook (Beezer, 2017) in the Spring semester of 2018 and (2) the various documents that instructors and students created as they used the textbooks. The textbook includes common linear algebra chapters (e.g., systems of linear equations, matrices, vector spaces, etc.).

Analysis of bi-weekly log data

The analysis of the viewing data revealed, unsurprisingly, that viewing tended to occur during the days when the classes were offered (mostly during class sessions), close to exams days, or when homework was due. The students mainly used solutions of exercises—in 17,405 viewings, 81% of the viewing time was for solutions of exercises, 15% for examples, and 5% for all the other elements. In the log responses students reported that they checked the textbook the day before class or the last day of their break; they also used it to study for the upcoming class, or when they were stuck, missed class, or had not understood their instructor's explanation. Students reported using mainly problems, exercises, and examples as they were preparing for class. When asked about their use of theorems, definitions, and examples, students said those were mainly used when producing notes for later use because they wanted to make sure they were connecting ideas and knew the basic definitions.

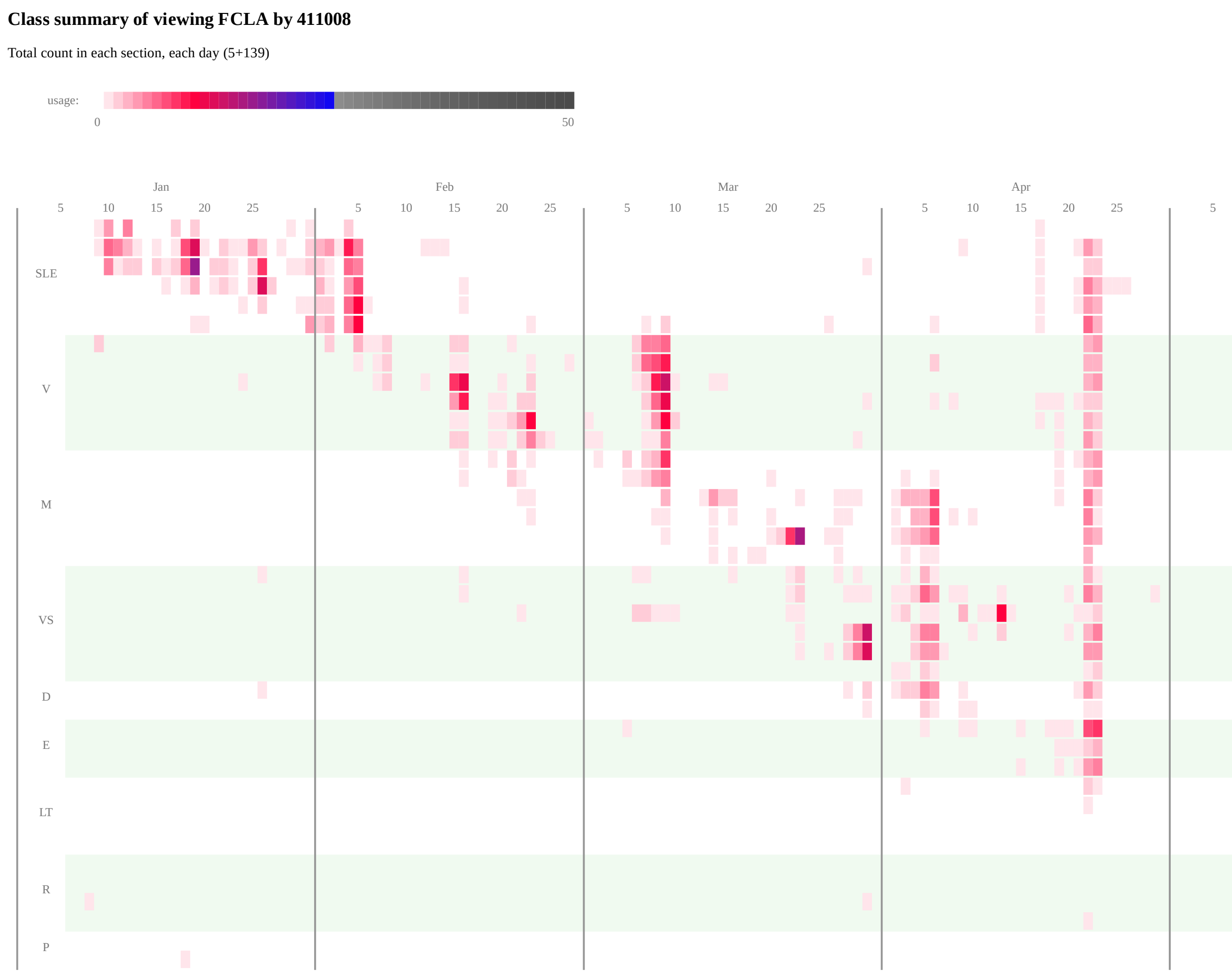

Figure 4. Viewing data for a course using the linear algebra textbook.

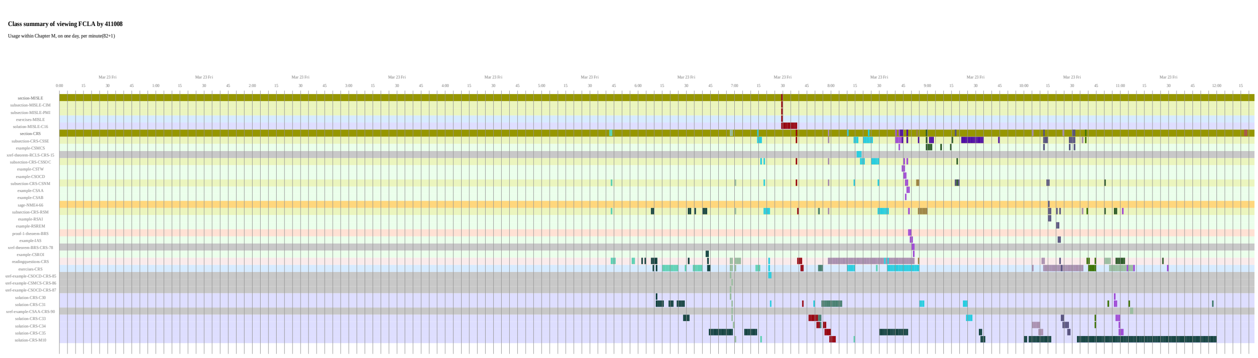

Figure 5. Viewing data at the individual level for a course using the linear algebra textbook.

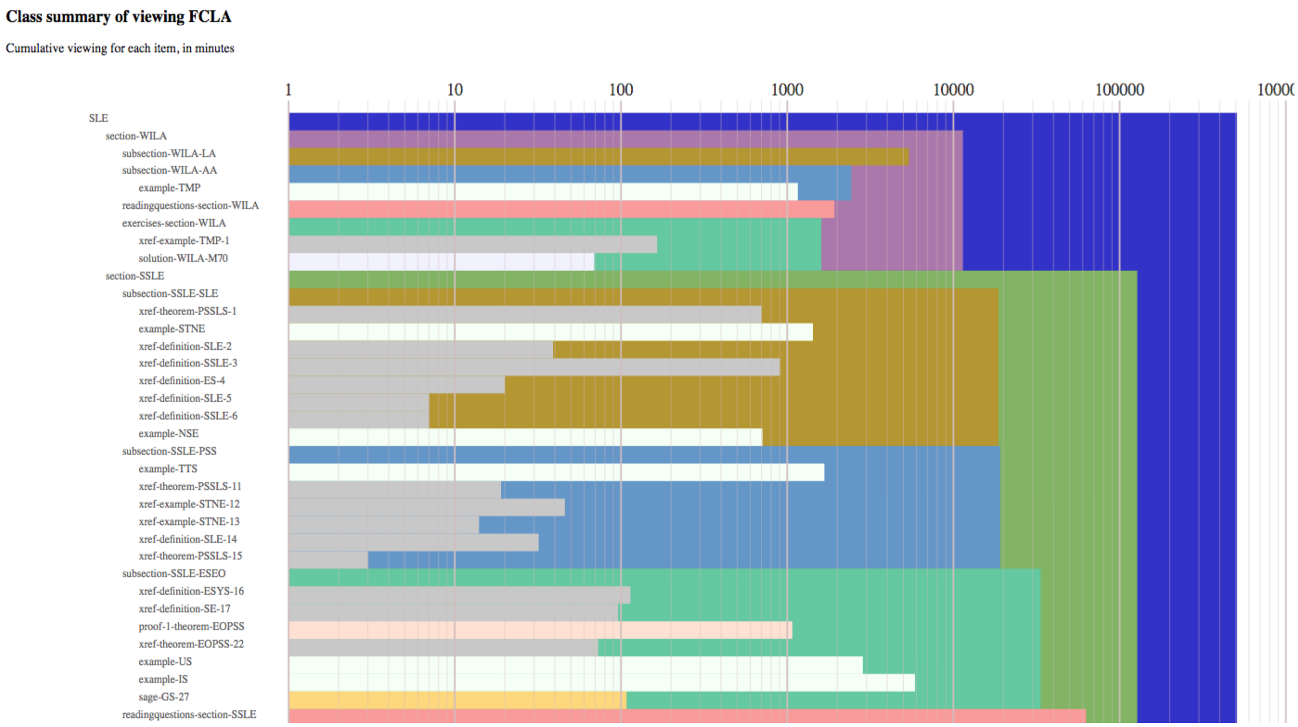

Figure 6. Data collection over a full term.

We found six themes in the analysis of student responses to their viewing of the linear algebra textbook (n = 120). The students indicated:

- Theme 1: viewing the textbook to study the material (58%); in addition, they mentioned some ways in which they do so. Students said that they:
- Theme 2: Start viewing examples (skipping text); then attempt the examples; and if need be, view the text to clarify ideas (17%).
- Theme 3: Start viewing homework; if the solution is wrong, read solution in detail to understand what problems they had (10%)
- Theme 4: Viewing to read word for word (8%)
- Theme 5: Viewing to cross-reference with class work (5%)
- Theme 6: Viewing formulas and definitions and keep track on personal document (3%)

Here are some illustrative examples:

- “I normally spend a couple of hours studying the book, I don’t have a set schedule or timetable for my studying. I also use my textbook to review and touch up on things I haven't looked at in a while.” (Theme 1)
- “Feb 19 Mon, Chapter M 6-9 PM: I was looking over example problems to help me prepare for my quiz, and I looked at definitions and theorems to help explain notation and solutions.” (Theme 2)

Analysis of documents

Instructors created lecture notes, syllabi, personal notes, and assessments, all with the goal of facilitating their teaching of the course. Students created class notes, homework documents or solutions, and textbook notes in order to improve their understanding, for practice, and reminders or memorization. Both students and instructors used many other resources. In terms of the instructors, they relied on colleagues, past notes, notes from when they were students, other textbooks, Wolfram alpha and other mathematical programs such as Sage, Maple or Mathematica, the textbook authors, and programming software, such as Python. Students mentioning working with classmates, the Internet, Google, YouTube, Chegg, Khan Academy, class lecture videos, other printed and HTML textbooks, family members, and their instructors. Students did not use the open-source feature and infrequently used computational cells.

Figure 7 summarizes the instrumentation and instrumentalization processes for the document lecture notes that we have found. They range from the less to more dynamic uses; the figure also highlights how instructors made use of their textbook.

Less to More Dynamic

Lecture Notes	Connection to the Textbook
Handwritten notes in paper (from points of reference to full notes)	References to the textbook
Online videos using the textbook	• Whole parts of the textbook • Practice problems from the textbook in accompanying problem sheets
Beamer/Power Point presentations	Hyperlinks to the textbook
Sage worksheets	• Hyperlinks to the textbook • Capabilities for the production of graphs and calculations of the textbook

Figure 7. Variation in use of the document *Lecture notes*, from less to more dynamic, and connections to the textbooks.

During classroom, some instructors copied their notes on the blackboard, whereas some distribute them ahead of time to the students, either as PowerPoints that they could annotate by printing them, or as Sage worksheets that students could manipulate in real time. The rules of actions and the reasons students and instructors had to use various documents are given in Figure 8.

	Rules of Action	When/Why
Students	"Read"	Study for examinations/class (study notes)
Students	Look for definitions	Clarify meaning to work out homework (homework solution)
Students	Study examples/proofs	Work out the homework (homework solution)
Instructor	Identify major course topics	Create syllabus before the term starts
Instructor	Identify theorems and definitions	Create lecture notes to be consistent
Instructor	Identify examples	Clarify definitions/theorems in class (lecture notes) Visualize definitions (lecture notes)

Figure 8. Rules of action and reasons for using documents by students and instructors.

Discussion

In general, the students and instructors seemed reluctant to take full advantage of novel features (such as the programming cells) or the reading questions. We speculated that by itself, the design of the textbooks is insufficient for facilitating the adoption of different ways of using these textbooks in teaching and learning. We noticed that students did not use features that were not required by their instructors and that they used those that their instructors said were important to use (e.g., definitions, theorems, examples, proofs).

Instructors might need training about ways to take advantage of the open nature of the textbooks. Some instructors, for example, only associated open source with the free access of the textbooks. We are planning gatherings and conversations with designers, authors, and instructors, so that the process of creating the textbooks becomes more transparent. Textbook production is expensive, and thus, research that documents how open access textbooks can be made widely available is important. Yet, without knowing how to best take advantage of the new technologies, we might not realize their potential within mathematics classrooms.

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